Laminar-Mixed Convection Heat Transfer from an Isothermal Inclined Flat Plate to Power Law Fluids

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Heat transfer due to the interacting effect of forced and free convection is not uncommon in real circumstances. The awareness that density differences leading to free convection currents in a forced convection situation could have detrimental effect in the design of heat exchangers has led to the recent consideration of mixed convection problems. For Newtonian fluids, laminar-mixed convection flow on flat plates has been studied for the plate being vertical, horizontal and inclined as can be seen from the works of Oosthuizen and Hart (1973), Chen et al. (1977), and Mucoglu and Chen (1979). For non-Newtonian fluids, the situation is not the same and only the vertical flat plate problem has been dealt with to date (Kubair and Pei, 1968; Shenoy, 1980; Lin and Sih, 1980).

The problem of mixed convection from an inclined vertical plate to a non-Newtonian fluid remains to be solved and this has been done in this paper. The Newtonian counterpart is available from the work of Mucoglu and Chen (1979) who have used the local nonsimilarity method to obtain a numerical solution. Here, however, the approximate method along the lines of Shenoy (1980) is used to estimate the heat transfer during laminar-mixed convection from an inclined plate to a power law fluid and the results so obtained are shown to compare well for Newtonian fluids with the numerical solution of Mucoglu and Chen (1979). The ease with which the solution is obtained more than compensates the loss in degree of accuracy of the solution.

THEORETICAL ANALYSIS

For laminar-mixed convection heat transfer to a power law fluid from an isothermal inclined flat plate, it will be assumed that the following equation suggested by Shenoy (1980) for non-Newtonian fluids and proposed by Churchill (1977) and Ruckenstein (1978) for Newtonian fluids in the vertical flat plate case holds good with appropriate definitions of $Nu_{x,F}$ and $Nu_{x,N}$ taking into account of the inclination of the plate. Thus

$$Nu_{x,M}^3 = Nu_{x,F}^3 + Nu_{x,N}^3 \qquad (1)$$

For pure forced convection flow, it is reasonable to assume that the inclination of the plate has no effect on the heat transfer and hence the expression for Nusselt number as given by Acrivos et al. (1960) can be used.

For pure natural convection, the angle of inclination would affect the heat transfer and one would have to obtain an expression for Nusselt number based on the following simplified forms of the governing boundary layer equations for laminar natural convection flow of power law fluids past a flat plate inclined at an acute angle of γ from the vertical.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty)\cos\gamma + K\frac{\partial}{\partial y}\left[\left|\frac{\partial u}{\partial y}\right|^{n-1}\left(\frac{\partial u}{\partial y}\right)\right] \qquad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad (4)$$

The simplifications involved in the derivation of the above equations have been done along the lines of Mucoglu and Chen (1979) and under the following conditions.

$$\tan\gamma \ll \frac{Gr_x^{1/2+n}}{\eta_\delta} \qquad (5)$$

where

$$Gr_x = \left(\frac{\rho}{K}\right)^2 [g\beta(T_w - T_\infty)]^{2-n} x^{2+n} \qquad (6)$$

$$\eta_\delta = \delta\left[\left(\frac{\rho}{K}\right)^{2/2+n} \{g\beta(T_w - T_\infty)\}^{2-n/2+n}\right] \qquad (7)$$

For Newtonian fluids, it can be seen that condition 5 is valid as typical realistic values of $Gr_x \sim 10^3$ and $\eta_\delta \sim 10$ for angles γ less than 45 degrees. The validity of the approximation increases with increasing Gr_x. In the case of power law fluids, it can be easily seen that the validity is further increased for lower values of n, i.e., increasing pseudoplasticity.

Equations 2–4 can be seen to be the same as the starting equations of Shenoy and Ulbrecht (1979) for laminar natural convection flow of a power law fluid past an isothermal vertical flat plate except for the term $\cos\gamma$ which brings in the contributions of the angle of inclination of the plate. Thus, the final expression for $Nu_{x,N}$ would also be the same except for an additional term of $\cos\gamma^{1/3n+1}$. Using the appropriate expressions for $Nu_{x,F}$ and $Nu_{x,N}$ the expression for laminar mixed convection heat transfer to a power law fluid from an isothermal flat plate inclined at an acute angle of γ from the vertical can thus be written as follows:

$$\frac{Nu_{x,M}}{Re_x^{1/n+1}} = \left\{\left(\frac{1}{0.893}\right)^3 \frac{1}{18}\frac{(2n+1)}{(n+1)}\left[\frac{117}{560\,(n+1)}\right]^{1/n+1} Pr_{x,F} + 8\left[\frac{1}{2}\left(\frac{3}{10}\right)^{1/n} f(n)\frac{(2n+1)}{(3n+1)} Pr_{x,F}\right]^{3n/3n+1} \times \left[\frac{Gr_x \cos^{2-n}\gamma}{Re_x^2}\right]^{3/(3n+1)(2-n)}\right\}^{1/3} \qquad (8)$$

Where

$$Re_x = \frac{\rho U_\infty^{2-n} x^n}{K} \qquad (9)$$

$$Pr_{x,F} = \frac{1}{\alpha}\left(\frac{K}{\rho}\right)^{2/n+1} x^{1-n/n+1} U_\infty^{3(n-1)/n+1} \qquad (10)$$

$$Pr_{x,N} = \frac{1}{\alpha}\left(\frac{K}{\rho}\right)^{2/n+1} x^{n-1/2(n+1)}[g\beta(T_w - T_\infty)]^{3(n-1)/2(n+1)} \qquad (11)$$

$$f(n) = \frac{1}{15} - \frac{5}{126n} + \frac{1}{84n^2} - \frac{1}{486n^3} + \frac{1}{5{,}103n^4} - \frac{1}{124{,}740n^5} \qquad (12)$$

Note that since the individual limiting expressions for $Nu_{x,F}$ and $Nu_{x,N}$ in the case of power law fluids were obtained under the high Prandtl number assumptions by Acrivos et al. (1960) and Shenoy and Ulbrecht (1979), the validity of Eq. 8 would be restricted to high Prandtl number situations which are known to occur for non-Newtonian fluids with high consistencies. Note also that for $\gamma = 0$, the equation for the vertical flat plate as given by Shenoy (1980) is recovered. The results of the present analysis for the inclined flat plate are borne out by Figures 1 and 2.

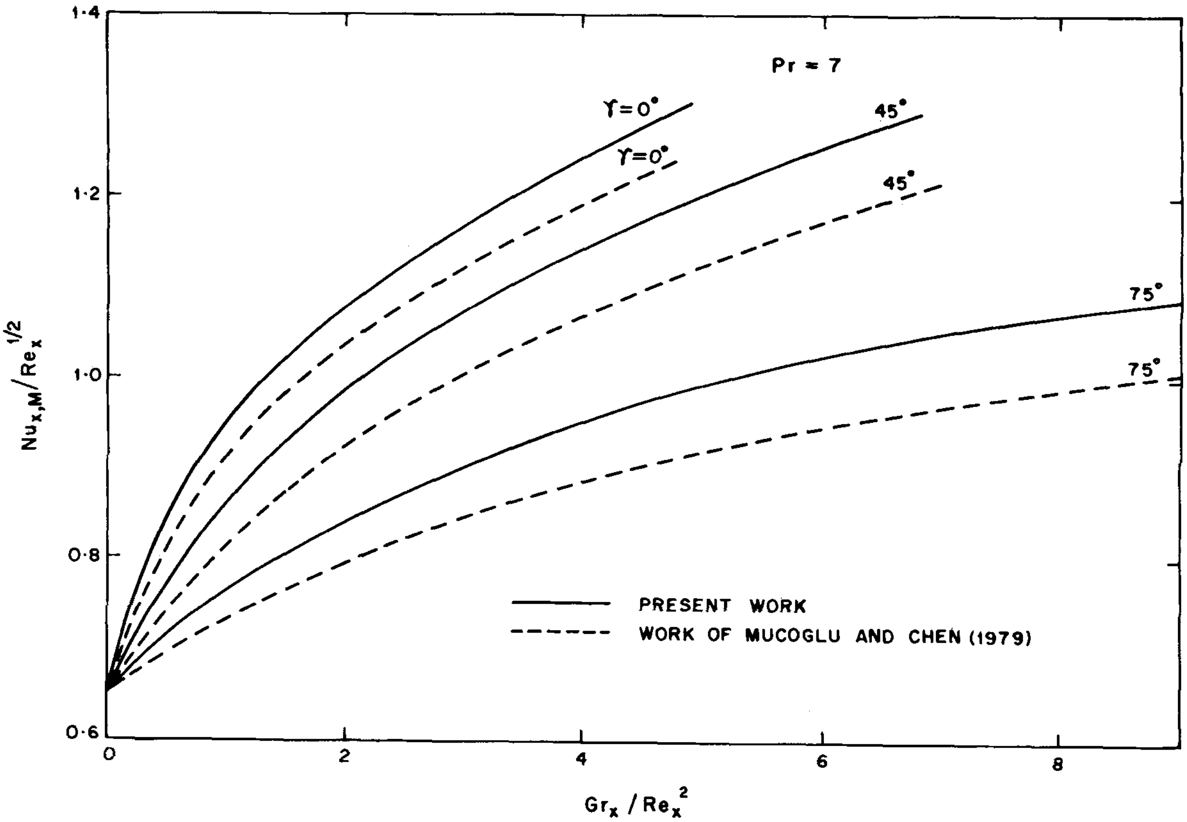

Figure 1. Comparison of this work with the predictions of Mucoglu and Chen (1979) for the effect of angle of inclination on the local Nusselt number in mixed convection.

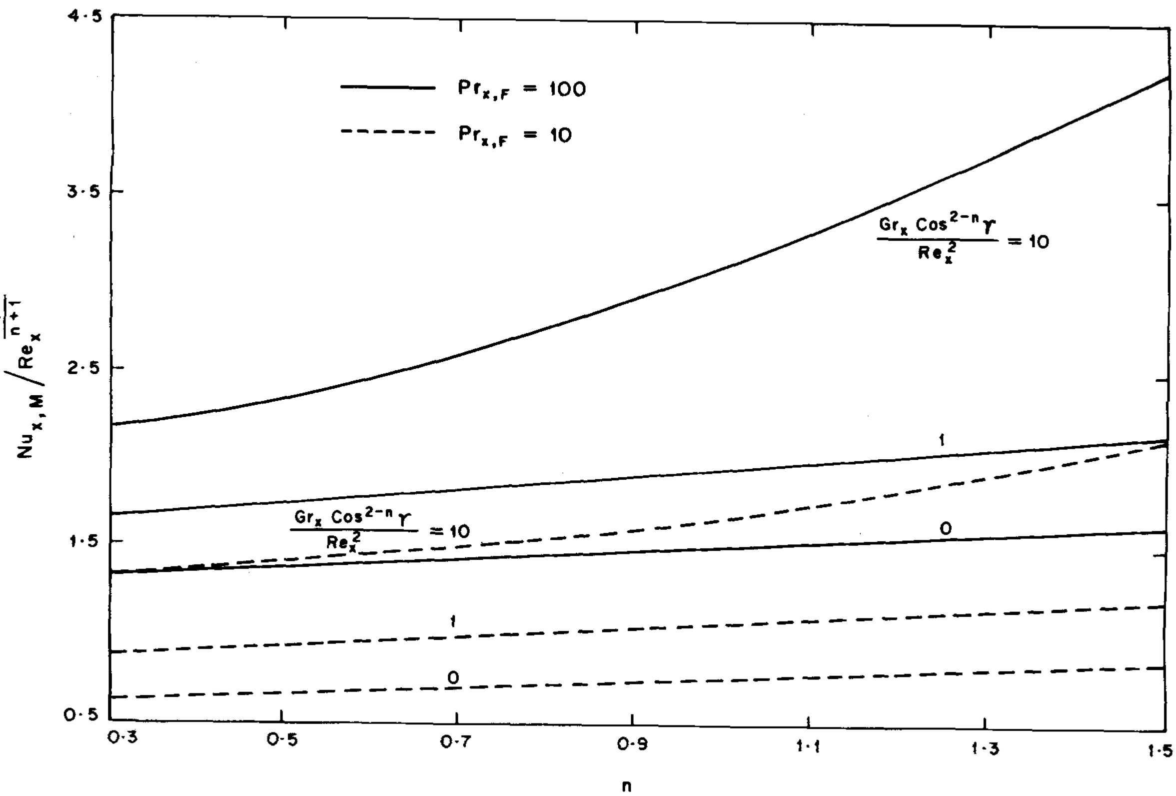

Figure 2. Variation of the local heat transfer rate with pseudoplasticity index n for laminar mixed convection past on isothermal inclined plate.

RESULTS AND DISCUSSION

Figure 1 compares the results of the present approximate analysis with those of Mucoglu and Chen (1979) for Newtonian fluids on a flat plate with varying angles of inclination. It must be noted that the comparison is made for $Pr = 7$ though the validity of the present results is limited to higher Prandtl numbers (>10). The propriety of the present work can be adjudged from the fact that the maximum deviation from the predictions of Mucoglu and Chen (1979) occurs at $Gr_x/Re_x^2 = 10$ and $\gamma = 75°$ and is only around 5%. The results of the present analysis would obviously give better predictions at higher Prandtl numbers which are commonly encountered in the case of non-Newtonian fluids.

Figure 2 brings out the effects of the various parameters in Eq. 8. It is found that the Nusselt number decreases with increased pseudoplasticity, i.e., lower n. The effect of buoyancy in the present case is to assist forced convection and thereby increase the Nusselt number with a more profound effect for dilatant fluids, i.e., $n > 1.0$ than in the case of pseudoplastic fluids, i.e., $n < 1.0$. The increase in heat transfer rate is greater at higher Prandtl number. The effect of angle of inclination is evident from Figure 1 which shows that the Nusselt number decreases with increasing departure from the vertical. Due to lack of experimental work on laminar mixed convection to power law fluids, a comparison between the predictions of the present analysis and experimental data cannot be made at the present time.

NOTATION

$f(n)$ = function of n defined by Eq. 12
g = acceleration due to gravity
Gr_x = generalized local Grashof number defined by Eq. 6
K = consistency index for a power law fluid
n = flow behavior index for a power law fluid
$Nu_{x,F}$ = Nusselt number based on local distance x for forced convection heat transfer to power law fluids
$Nu_{x,M}$ = Nusselt number based on local distance x for mixed convection heat transfer to power law fluids and defined by Eq. 1
$Nu_{x,N}$ = Nusselt number based on local distance x for natural convection heat transfer to power law fluids
$Pr_{x,F}$ = generalized Prandtl number based on local distance x for forced convection heat transfer to power law fluids and defined by Eq. 10
$Pr_{x,N}$ = generalized Prandtl number based on local distance x for natural convection heat transfer to power law fluids and defined by Eq. 11
Re_x = generalized Reynolds number based on local distance x for forced convection to power law fluids and defined by Eq. 9
T = temperature of the fluid
T_w = temperature of the plate
T_∞ = temperature of the bulk of the fluid
U_∞ = free stream velocity for forced convection
u = velocity component along the x coordinate
v = velocity component along the y coordinate
x = distance along the plate from the leading edge
y = distance normal to the plate

Greek Letters

α = thermal diffusivity of the fluid
β = expansion coefficient of the fluid
γ = angle of inclination from the vertical
η_δ = dimensionless boundary layer thickness defined by Eq. 7
ρ = density of the fluid

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Vapor-Liquid-Liquid Equilibria using UNIFAC in Gasohol Dehydration Systems

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The UNIFAC group contribution method has been used to model the nonideal liquid behavior in the system water(1)–ethanol(2)–cyclohexane(3). The VLLE data for a given feed composition lies in a narrow temperature range. Outside of this range are neighboring LL and VL regions. Similar complex equilibria can be expected in gasohol dehydration systems using azeotropic distillation.

INTRODUCTION

The UNIFAC group contribution method developed by Fredenslund et al. (1975) has been widely used to calculate vapor-liquid equilibria (VLE) for systems with nonideal liquid phases. The UNIFAC tables have been steadily increased by Skjold-Jørgensen et al. (1979) and Gmehling et al. (1982). The UNIFAC-